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# An econometric model of conventional mortgage credit flows and the channeled effects of monetary policy

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An econometric model of conventional mortgage credit flows and the channeled effects of monetary policy

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# Daniel E. Laufenberg

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major: Economics

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Signature was redacted for privacy.

In Charge of Major Work

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#### I. INTRODUCTION

Recently, strong concern has been displayed over the status of the housing sector in the United States. As a result, this study has been undertaken to deal with one of the problems significant to that status. It is apparent that the mortgage credit market is important to the housing sector, since the terms and availability of mortgage credit have a direct bearing on user demand for housing accommodations (25). Therefore, the major concern in this study is not the housing sector per se, but how and to what degree the discretionary tools of monetary policy affect the terms and availability of mortgage credit, thus implying the influence of monetary policy on the housing market. To present a study of this type, it is necessary to derive an econometric model of the mortgage credit market, consisting of functional relationships of the demand for and supply of mortgage credit.

A comprehensive model of this type would be beyond the scope of the study presented here. The solution is to restrict the analysis to a specific area of the mortgage credit market. To accomplish this, the problem has been limited to isolating the effects of discretionary mometary policy on the demand for and supply of mortgage credit from the savings and loan associations (S & L's) for single-family, owner-occupied, conventionally financed homes.

The plan of the paper is as follows. The second chapter is confined to developing the demand for conventional mortgage credit under the assumption that the individual attempts to maximize his wealth. It is from the first order conditions of the Lagrangian expression of this constrained

maximum that the variables to be used in the demand function are determined. The third chapter deals with the formulation of the supply of conventional mortgage credit from a savings and loan association. Again, a constrained maximum, but of S & L profits, is used to determine the supply function. A short note on the difficulties encountered in aggregating the model for empirical analysis is included at the end of chapter three. Chapter four presents the formulation of an adjustment mechanism that will take into consideration the importance that demand plays in determining the equilibrium rate of interest on conventional mortgages. In chapter five, the channels of monetary policy are incorporated into the model.

Monetary forces will be assumed to operate in the mortgage credit market through three channels. The channels are the cost-of-capital effect, the wealth effect, and credit rationing. Many econometric studies have taken into consideration only the cost-of-capital effect of monetary policy, thus often causing the results to be biased against monetary forces. This bias has caused concern, especially with respect to the housing sector, because even though these studies have failed to show the full impact of monetary policy on the economy, the fluctuations that occur in the housing market are often attributed to monetary forces (8). In an attempt to eliminate this bias, the two additional channels of monetary policy will be included in this model.

The first channel of monetary policy, the cost-of-capital effect, has its impact on the buy-or-rent decision of firms and individuals. If the cost of owning capital is high relative to the cost of renting, then fewer units of capital are purchased. If the cost of owning capital is low relative to the cost of renting, the opposite is true. Therefore, the implicit

cost of owning capital is just as important as the explicit cost of renting capital when the decision to buy-or-rent capital services is made. For the purposes of this study, the cost-of-capital influences of monetary policy will be most directly related to the decision of the individual to buy-orrent housing services.

The second channel of monetary policy is the wealth effect. This effect implies that as a result of monetary policy, the value of the assets held in the portfolios of individuals and financial intermediaries change. The change in the value of the assets will cause the wealth of the individual and the net worth of the firm to change, thus leading to changes in the consumption of goods and services, including the services provided by owner-occupied housing to the individual.

With respect to the mortgage credit market, the third channel of monetary policy, credit rationing, suggests that nonprice rationing occurs when the rate of interest on mortgage credit is slow to adjust to the excess demand for mortgage credit that may exist as a result of monetary forces (6), (8), and (19). As a result, the credit that is available is rationed on terms other than price considerations.

It is through these three channels that monetary policy is assumed to operate; it will be through these three channels that the effects of discretionary policy on the mortgage credit market for single-family, owner-occupied, conventionally financed housing units, hereafter referred to as simply the mortgage credit market, are examined. The cost-of-capital effect and wealth effect are assumed relevant to the demand for mortgage credit, whereas credit rationing and, indirectly, the wealth effect are assumed relevant to the supply of mortgage credit. Chapter six will deal with the empirical results and conclusions of the complete model.

#### II. REVIEW OF THE LITERATURE

#### A. Points of View

Although similarities between this study and past econometric studies undoubtedly exist, I believe this paper makes a twofold contribution: the theoretical approach taken in developing a model of the mortgage credit market; and the attempt to directly relate discretionary monetary policy to this market through the three channels of monetary policy. A review of past studies provides a basis for comparing these contributions to the preceding literature.

The three most recent studies that have been done in the area of mortgage markets have been papers by Brady (4), Huang (14), and Jaffee (16). Of these three studies, Brady's does not deal directly with the mortgage credit market, but is instead primarily concerned with new housing starts. In focusing on housing starts, however, Brady has made implicit assumptions about their relationship to the mortgage market. He also distinguishes between the types of financing involved in housing starts, i.e., conventional housing starts, FHA-insured housing starts, or VA-guaranteed housing starts. Huang makes the same kind of distinction with respect to the type of mortgage credit, whereas Jaffee separates the mortgage market into different sectors based on the institutional source of the mortgage credit. Some of both features will be used in this study, since the primary problem is structural estimation of an econometric model of conventional mortgage credit flows from S & L's.

# B. The Demand Side

In Brady's <u>a priori</u> functional relationship of the demand for housing starts, it is assumed that "net household formation, interregional migration, housing demolitions and per capita real disposable income should all be positively related to the demand for residential housing and that as the relative price of housing services declines, the demand for housing rises" (4, p. 4). The statistical results of Brady's model led him to conclude that the variables on the demand side of the market were not significant, and that "the basic demand variables dropped out" (4, p. 17). As a result, he concluded that housing starts depend on the supply of mortgage credit through the financial variables associated with that credit, and did not depend on demand (4), (12).

Huang does not contradict Brady's conclusion, but since Huang is dealing with a particular market in the financial sector, the mortgage credit market, he does provide a twist in the method used. His study is concerned with the demand for mortgage loan funds by the household sector and is postulated as a stock adjustment function. That is, the gross demand for mortgage credit is a function of the proportional difference between the desired level of mortgage debt, which is treated as a function of the desired stock of housing and the interest rate on mortgage borrowing, and the opening stock of mortgage debt. Similar to Brady's demand function above, Huang assumes the desired stock of housing to be a function of disposable personal income and the ratio of the rent component of the consumer price index and the Boeckh index for the cost of residential construction. But unlike Brady, Huang includes in the demand for mortgage

credit the credit terms of the loan-to-value ratio, the length of amortization period, and the contractual interest rate in the demand for mortgage credit function. The contractual interest rate is replaced by the market mortgage yield, which Huang considers the relevant cost-ofborrowing variable. This variable has the expected negative effect when the demand function is estimated with two-stage least-squares. In studies by Sparks (26) and Silber (23), the yield coefficient is positive and statistically significant, while in the studies by Brady (4) and Jaffee (16) the sign of the yield variable is difficult to determine.

Huang combines the two noninterest credit terms into one composite variable called the "percentage of loan to value paid per annum" which is obtained by dividing the loan-to-value ratio by the average length of loan amortization (14, p. 15). He gets the expected negative coefficient on this variable.

Another variable in Huang's demand function is included to take into consideration the short-run financial decision of the household sector. Because there is some statistical evidence that households accumulate liquid assets in anticipation of expected large expenditures (31), and the relatively low cost of savings withdrawals for these expenditures, the ratio of the increase in savings accounts to net acquisition of capital market instruments by households is the variable that Huang chooses to measure the adjustment in the portfolios of households.

Jaffee states that the demand for mortgage credit by the household is related to the theory of portfolio choice. "The basic variable in such a formulation, at a given point in time, is the household's level of net worth" (16, p. 39). The household will distribute its total net worth among various financial and real assets in order to maximize some utility

function, where the utility function takes into consideration the services provided by the real assets, the interest income obtained from financial assets, and liquidity value of both types of goods. If the possibility of borrowing is introduced into the decision process, then the households' ability to obtain credit will depend upon some type of collateral requirement. Therefore, the demand for mortgage credit by households will depend on the stock of houses. A stock adjustment process is introduced to consider the change in the demand for mortgage credit. The speed of adjustment to the desired level of mortgage debt will depend on the relative cost of mortgage financing, where the alternative is a weighted average of rates on other instruments.

#### C. The Supply Side

Although most past studies have derived the demand functions as a result of <u>a priori</u> knowledge of the maximizing conditions of consumer behavior or portfolio management, little has been done by way of attempting to explain the supply functions on a similar basis. Evidently, most studies implicitly assume the supply function to be the result of the maximizing behavior of the financial institutions involved in the mort-gage credit market.

The supply function for housing starts formulated by Brady in his working paper includes many of the financial variables that have appeared in the demand functions for mortgage credit of other studies. It seems that the <u>a priori</u> formulation of the supply function by Brady follows from the Guttentag conclusion that fluctuations in the supply of mortgage credit were more important than fluctuations in demand in determining

fluctuations in residential construction (12). But while Brady's <u>a priori</u> supply function reflects this conclusion, at the same time it includes variables that appear to be relevant to the demand for mortgage credit such as the interest rate on mortgages, amortization periods, loan-tovalue ratios, and residential construction costs.

Huang provides a supply function that includes the net savings inflow into bank and nonbank financial intermediaries, the yields from nonmortgage capital market instruments as proxies for the opportunity cost of mortgages, the rate of interest on mortgages, and a variable to measure the effect on the mortgage market of monetary policy which will be discussed later. These are basically the same variables that are included in Jaffee's supply function, although two distinguishing characteristics separate the studies. First, Jaffee's supply function is assumed to be the result of the intermediaries' efforts to manage their portfolios with respect to the net yields and liquidity value of the assets. Huang makes no assumptions about the behavior of the intermediaries but does provide explanations for the variables he includes in the supply function. The second characteristic is that Jaffee uses a stock adjustment model with respect to the mortgages the intermediaries desire to hold plus the net inflow of deposits, and mortgage repayments to these intermediaries, as variables of mortgage credit supplied. The desired level of mortgages that the actual level is adjusted to is a function of the difference between the mortgage rate of interest and the interest rate on a single capital instrument. Jaffee realizes that mortgages are substitutes for many capital instruments but for statistical purposes has eliminated all

but one. Huang does use an adjustment model for the demand side of his model but makes no attempt to formulate the supply function in a similar manner.

D. Adjustment

When a study of the mortgage credit market is undertaken, one problem that is encountered is measuring the credit rationing effect on this particular market. As a result, most recent studies (Huang (14) and Jaffee (16)) have included a mortgage interest rate adjustment mechanism to take into consideration the lag of adjusting the interest rate to the equilibrium rate. Introducing an adjustment mechanism, however, causes problems of identification and estimation. These arise basically because: "(1) the time-series data on the supply of mortgage credit and the demand for mortgage credit are identical and these variables are both endogenous in the market context, and (2) the existence of credit rationing is rather difficult to capture statistically" (14, p. 17).

When allowance is made for a disequilibrium mortgage rate, assumpiions must be made to alleviate those conditions that cause the estimation problem. Huang assumes that the observed points fall on the demand curve rather than the supply curve, so that the difference between the first-stage estimate of mortgage credit supplied and the actual observation gives a measure of the excess supply. He also includes the change in the noninterest credit terms variable to consider the credit rationing.

Jaffee's assumptions are more difficult to follow. His empirical results indicate "that the supply of mortgage (credit) is relatively unresponsive to the current mortgage rate because of the reliance on

advance commitments" (16, p. 42). As a result, in the short-run the mortgage rate is assumed to adjust such that it brings the current mortgage credit demand into line with the predetermined supply. Because commitments are of primary importance to the supply side, the observed mortgage flows were considered the demand variable in the adjustment mechanism.

Credit rationing is implied to exist in the market by allowing for an adjustment of the interest rate to the equilibrium rate. If this rate of adjustment is assumed to be less than one, then credit-rationing is the alternative, i.e., the use of noninterest credit terms in allocating the mortgage credits available to alternative uses.

# E. Monetary Effects

Most recent econometric models of the mortgage market have incorporated the effects of monetary policy by the use of an "index of monetary policy". The problem is, that the index is, in most cases, applied only to the supply side of the market. Also, there has been no attempt to separate the effects of monetary policy on the mortgage credit market. This is unusual, since fluctuations in this market are so closely associated with fluctuations in the housing sector where monetary policy is assumed to have such an immediate and pronounced effect. Brady includes an index of monetary stringency as one of the supply variables of his model but, as such, it is difficult to follow its effect on the market. He does, however, consider what he calls the "Maisel-Brownlee Controversy". Brownlee concludes that fluctuations in residential construction activity are a result of the high interest elasticity of demand for residential housing. Thus during periods of monetary stringency, the high interest

rates will cause a decrease in demand for housing. Maisel concludes that housing is determined primarily by supply variables, since it is dependent on the supply of mortgage funds. Monetary stringency widens the spread between capital market rates and the regulated rate paid on deposits of financial intermediaries. This causes deposits to fall and thus the supply of funds available for mortgages to decrease. Brady's empirical work supports the Maisel hypothesis.

Huang also considers the monetary effects on the mortgage credit market by analyzing the time lag between monetary variables, such as the monetary base, and mortgage credit flows and yields. Some of the inferences that are made from the results are that (a) the peak effect of the monetary base on mortgage credit flows is manifested during the current quarter and (b) when the monetary base expands, the mortgage yields initially fall, but after six months the yields rise. Therefore, along with the credit rationing effect implied by the adjustment mechanism, the monetary base is included in the supply function of mortgage credit. Similar to Brady, Huang does not consider monetary effects on the demand side of the mortgage market.

In the third study, Jaffee allows for credit rationing by including an adjustment mechanism in his model of the mortgage market. Such a mechanism makes possible a disequilibrium mortgage rate. However, since this model is only a part of a much larger simultaneous model, no other channel of mometary policy is explicitly outlined. One set of estimates of the model presented by de Leeuw and Gramlich (8), suggested that the only channel of mometary policy that was evident in the housing sector was that of credit rationing.

#### F. Comparison

There are four major differences that exist between the econometric models that have been reviewed above and the econometric model that is to be developed in this study. The first difference is that monetary policy is assumed to have three channeled effects on the mortgage credit market. They are the cost-of-capital effect, the wealth effect, and the credit rationing effect.

Second, monetary policy affects not only the supply of mortgage credit but also the demand. The mortgage credit studies reviewed above have ignored the effects of monetary policy on the entire market. In order to be more comprehensive in measuring the effects of monetary policy, this study takes into consideration the channeled effects on the demand as well as the supply of mortgage credit.

The third difference is that mathematics is used to assist in the theoretical construction of the market equations. The calculus of variations provides a dynamic approach to the mortgage credit problem.

The adjustment mechanism incorporates the relationship between the nominal rate of interest on mortgage credit and the demand for mortgage credit. This is another difference between the other mortgage studies and this study of mortgage credit. Also, the adjustment mechanism allows for changes in the nominal mortgage rate of interest which may be caused by other than a disequilibrium condition in the market.

There may be additional comparisons that can be made between this and past studies, but these four differences are felt to be the major differences. They are felt to be important enough to warrant the presentation of the model, the empirical results, and the conclusions.

#### III. DEMAND FOR MORTGAGE CREDIT

#### A. Introduction

The equilibrium conditions of a household's accumulated wealth rest implicitly on maximizing behavior. "It so happens that in a wide number of econometric problems it is admissable and even mandatory to regard our equilibrium equations as maximizing (minimizing) behavior" (22, p. 22). From these conditions we can derive the desired level of assets that should be held by the household in its portfolio.

Moreover, it is possible to derive operationally meaningful restrictive hypotheses on consumers' demand functions from the assumption that consumers behave so as to maximize an ordinal preference scale of quantities of consumption goods and services (of course, this does not imply that they behave rationally in any normative sense) (22, p. 21).

Household behavior, with respect to its non-human wealth, will be determined by an attempt to maximize the returns it receives from the asset portfolio in every time period (maximizing net worth), subject to the substitutability of the assets which is taken into consideration by introducing an objective function of risk and return. It is assumed that the assets held are both real and financial (11). The real asset is housing and the financial assets are savings, equities, and bonds. Of these assets, only the real asset, housing, is a debt-financed asset. That is, any long-term debt accumulated during the period by the individual is directly proportional to the change in the amount of housing held. It is also assumed that these assets are substituted for one another but are less than perfect substitutes due to differences in the exchange costs, marketabilities, reversibilities, and negotiabilities, i.e., differences in

liquidity. With the purchase of each asset, there is associated with that asset some level of risk, and the risk associated with one type of asset is never the same as that associated with another type, but all assets of the same type have the same risk. Thus, the preference function depends upon the return and the risk of the assets included in the portfolio, and is treated as the objective function of the maximization problem. This "utility function" is assumed to satisfy the necessary conditions for an interior solution to exist.

The household will additionally take into consideration their human wealth when attempting to maximize the total wealth accumulated over time. The return on human wealth will be expressed by the wages and salaries of the household.

This is not an attempt to revise the theory of consumer behavior, but only an attempt to apply the theory to a particular problem.

There is, moreover, considerable advantage in discussing the problem first in its full generality. The high degree of abstract-ness will be more than compensated for in the ease with which numerous applications can be deduced as special cases (22, p. 23).

Some of the assumptions mentioned above may seem to be over simplifying, but to analyze with any precision a complex market such as that of mortgage credit, a certain degree of simplification is required. Hopefully, the approach taken to analyze the mortgage credit market will provide the means to obtain meaningful restrictive hypotheses.

B. Maximizing the Household's Wealth

The motives for holding wealth are outlined by Milton Friedman to include (1) the need for the household to maintain some even flow of

expenditures even though their receipts vary widely from time period to time period, (2) their desire to earn interest on loans when the interest rate is positive, and (3) the household will attempt to hedge against uncertainty by maintaining a reserve for emergencies (11). As a result, the household will attempt to maximize their wealth, given the constraints that are relevant to its optimizing decisions.

There are a number of different forms that wealth can take; the major general distinction is between human and nonhuman wealth (11). Human wealth is the element that provides the salary that the individual can command in the market. This implies that the marginal physical productivity of labor services from an educated and/or experienced worker is higher than that of an uneducated and/or experienced unit of labor. Therefore, as more time and money is invested into the labor unit, the greater the productivity and, as a result, the higher the salary (higher wage rate at same number of hours worked) he can demand in the labor market. The return on human capital is the salary (W(t)) of the household.

The other form of wealth is nonhuman wealth. All forms of nonhuman wealth are not equally satisfactory in the portfolio. Another major distinction that can be made is between real and financial assets of the nonhuman component of wealth.

The household attempts to hold in its portfolio the optimal levels of nonhuman assets, i.e., bonds, time deposits, housing, and equities, in such a way as to maximize the net receipts of the portfolio discounted over time. With the aid of the assumptions given at the beginning of this chapter, the function that represents the net receipts can be expressed as:

$$R(t) = r_{b}(t)B(t) + r_{h}(t)H(t) + r_{e}(t)E(t) + r_{s}(t)S(t) - c_{d}(t)D(t)$$
  
-  $R_{e}(t)$ ,<sup>1</sup>

where R(t) is the net receipts;  $r_b(t)$  is the yield rate on bonds; B(t) is the dollar value of bonds held in the portfolio;  $r_h(t)$  is the yield rate on housing; H(t) is the dollar value of housing held in the portfolio;  $r_g(t)$  is the yield rate on time deposits; S(t) is the dollar value of time deposits held in the portfolio;  $r_e(t)$  is the yield rate on equities; E(t) is the dollar value of equities held in the portfolio;  $c_d(t)$  is the costof-credit in the present period (t); D(t) is the amount of debt accumulated during the present period; and  $R_e(t)$  is the dollar value of amortized payments from debt accumulated in past periods.

The reason D(t) is used is because it takes into consideration the change in the stock of debt  $(\dot{L}(t))$  and the portion of past debt repaid during the present period ( $\theta L(s)$ ). The usual expression for net investment is gross investment minus replacement. If that principle is applied to the accumulation of debt by the household, the debt accumulated is equal to gross debt minus debt repaid. The expression would be:

 $\dot{L}(t) = D(t) - \theta L(s),$ 

which, when solved for D(t), becomes:

 $D(t) = \dot{L}(t) + \theta L(s).$ 

Because of the heterogeneity in the cost-of-credit over time, the present cost-of-credit should apply not only to the change in the stock of debt, but to any debt accumulated during period t. As a result, included in the

<sup>1</sup>A glossary of terms is provided at the end of this study.

expression above is the portion of past debt repaid, which, if relevant to the amount of debt accumulated during the present time period, is financed at the new cost-of-credit.

It should also be noted that repayments include both interest payments and repayment of the debt. This means that the repayments are equal to the following:

$$R_{e}(t) = c_{d}(s)L(s) + \theta L(s),$$

where  $c_d(s)$  is the old cost-of-credit, L(s) is the amount of debt outstanding at the beginning of the period, and  $\theta$  is the portion of that debt to be repaid. If repayments are canceled out by new accumulations of debt, that new debt will be financed at the new cost-of-credit, i.e., the present interest rate on mortgage credit. Therefore, the repayments must be included for a second time because they represent a liability.

Combining the two components, salary and net receipts, provides the income flow that the household will receive during a particular time period. Therefore, the objective function that is to be maximized is the income flow:

F(t) = W(t) + R(t).

Because of the uncertainty with respect to future flows, the elasticity of expectations with respect to the income flow is assumed equal to one, i.e., any percentage change in current income is assumed to cause an equal percentage change in expected income. Therefore, the wealth of the household is the discounted present value of the future expected income flow over a finite time period.

In addition to the present discounted value of the flow of funds that will result from the wealth of the household, some consideration must be

given to the increased risk that the household assumes when its portfolio consists primarily of only one asset. The preference for a particular asset is a function of both the return on that asset and the level of risk associated with holding more of that asset in the portfolio. Therefore, the risk element should be included in the objective function. This is accomplished by including in the expression an implicit cost function for the household that measures the additional risk that results from shifting between assets. The cost-risk function will be represented by the general form:

 $C_p = F(B/K, H/K, E/K, S/K),$ 

where  $C_R$  is the implicit cost associated with the risk of the portfolio, K is the total dollar value of the assets, and B, H, E, S are the dollar values of bonds, housing, equities, and savings, respectively. This particular function is assumed strictly convex, which will guarantee the concavity of the objective function--a necessary condition for an interior solution to the maximization problem. By specifying the cost-risk function in this manner, an additional assumption is implied about the household's behavior, i.e., it is a risk averter.

The constraints on the wealth function (F(t)) are that any debt accumulated in the period is used to finance housing and that the down payment for housing must come from wages, realized return on the portfolio of nonhuman wealth, and/or changes in the stock of other assets. The first constraint is that the amount of aebt accumulated during the period is proportional to the change in the stock of housing. Other than housing, no other asset in the portfolio is financed with debt. As a result, the constraint will be represented by:

 $D = \alpha \dot{H},$ 

where  $\alpha$  is the loan-to-value ratio and H is the change in the stock of housing in period t. It was stated earlier that  $D = L + \theta L(s)$  and if this expression is substituted into the one above, the result will be:

 $\dot{L} + \theta L(s) = \alpha \dot{H}.$ 

The above expression states that if there is a change in the stock of housing held by the individual, it is proportional  $(\alpha)$  to the gross of mortgaged debt accumulated during the period.

The second constraint is that the portion of the change in the housing stock held by the household not covered by the mortgage credit it receives must be financed by transferring the equity it has in other assets accumulated as part of its portfolio (primarily the savings account), the return on the portfolio that it may wish to use for the down payment, and that part of wages of the current period that is not used for consumption purposes. This constraint can be expressed as:

 $(1 - \alpha)\dot{H} = R - \dot{E} - \dot{S} - \dot{B} + W - C - R_{a},$ 

where R is the net receipts from the portfolio held;  $\dot{E}$ ,  $\dot{S}$ , and  $\ddot{B}$  are the changes in equities, savings, and bonds, respectively; W is the salary of the household; C is the level of consumption expenditures; and R<sub>e</sub> is the amortized payment of past debt.

Combining the objective function of wealth (F(t)) and the constraints relevant to the portfolio decisions of the household, the result is the following Lagrangian expression:

$$J = \int_{0}^{\infty} \left\{ e^{-rt} \left[ W + r_{b}^{B} + r_{h}^{H} + r_{e}^{E} + r_{s}^{S} - c_{d}^{D} - R_{e} \right] - F(B/K, H/K, E/K, S/K) + \lambda_{1}(t) \left[ D - \alpha \dot{H} \right] + \lambda_{2}(t) \left[ (1 - \alpha) \dot{H} - R - W_{e}^{H} \right] \right]$$

$$+C + R_{r} + \dot{E} + \dot{S} + \dot{B}$$
] dt. [3.1]

To simplify the notation, let everything within the braces be represented by f(t). The first-order conditions are as follows:

$$\frac{\partial f(t)}{\partial R_e} = -e^{-rt} + \lambda_2(t) = 0. \qquad [3.2]$$

$$\frac{\partial f(t)}{\partial B} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial B} \right) = r_{b} e^{-rt} - \frac{\partial F}{\partial B} e^{-rt} - \frac{d}{dt} [\lambda_{2}(t)] = 0.$$
 [3.3]

$$\frac{\partial f(t)}{\partial E} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial E} \right) = r_e e^{-rt} - \frac{\partial F}{\partial E} e^{-rt} - \frac{d}{dt} [\lambda_2(t)] = 0.$$
 [3.4]

$$\frac{\partial f(t)}{\partial S} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial S} \right) = r_{s} e^{-rt} - \frac{\partial F}{\partial S} e^{-rt} - \frac{d}{dt} [\lambda_{2}(t)] = 0.$$
 [3.5]

$$\frac{\partial f(t)}{\partial D} = -c_d e^{-rt} + \lambda_1(t) = 0. \qquad [3.6]$$

$$\frac{\partial f(t)}{\partial H} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial \dot{H}} \right) = r_{\rm h} e^{-rt} - \frac{\partial F}{\partial H} e^{-rt} - \frac{d}{dt} \left[ -\alpha \lambda_1(t) + (1 - \alpha) \lambda_2(t) \right] = 0.$$
[3.7]

$$\frac{\partial f(t)}{\partial \lambda_1(t)} = D - \alpha \dot{H} = 0.$$
 [3.8]

$$\frac{\partial f(t)}{\partial \lambda_2(t)} = (1 - \alpha)H + E + S + B - (R + W - C - R_e) = 0.$$
 [3.9]

The Legendre conditions of maximization are assumed to hold.

It can be shown from above that  $\lambda_2(t)$  is equal to  $e^{-rt}$  and that  $\lambda_1(t)$  is equal to  $c_d e^{-rt}$ . The first time derivatives of these variables are  $d\lambda_2(t)/dt = -re^{-rt}$  and  $d\lambda_1(t)/dt = rc_d e^{-rt} + e^{-rt}c_d$ . Substituting these results into the first-order conditions above, the new expressions are:

$$\frac{\partial f(t)}{\partial H} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial H} \right) = r_{h} e^{-rt} - \frac{\partial F}{\partial H} e^{-rt} - \alpha [rc_{d} e^{-rt} - e^{-rt}c_{d}]$$

+ 
$$(1 - \alpha)re^{-rt} = 0.$$
 [3.10]

$$\frac{\partial f(t)}{\partial B} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial B} \right) = r_{b} e^{-rt} - \frac{\partial F}{\partial H} e^{-rt} + r e^{-rt} = 0.$$
 [3.11]

$$\frac{\partial f(t)}{\partial E} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial E} \right) = r_e e^{-rt} - \frac{\partial F}{\partial E} e^{-rt} + re^{-rt} = 0.$$
 [3.12]

$$\frac{\partial f(t)}{\partial S} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial S} \right) = r_{s} e^{-rt} - \frac{\partial F}{\partial S} e^{-rt} + r e^{-rt} = 0.$$
 [3.13]

The partial derivatives  $\partial F/\partial B$ ,  $\partial F/\partial E$ ,  $\partial F/\partial H$ , and  $\partial F/\partial S$  are equivalent to the marginal risk of bonds, equities, housing and savings, respectively. Therefore, the rate of substitution (RS) between housing and any other asset held in the portfolio, e.g., bonds, is equal to the ratio of the implicit rates of return:

$$\frac{\partial F/\partial B}{\partial F/\partial H} = r_b * / r_h * = RS, \qquad [3.14]$$
  
where  $r_b * = (r_b + r)e^{-rt}$  and  $r_b * = [r_b - \alpha rc_d + \alpha \dot{c}_d + (1 - \alpha)r]e^{-rt}$ .

Because of the debt financing involved, the implicit rate of return on housing  $(r_h^*)$  is not simply the yield rate plus the rate of discount like on the other assets included in our model, but involve a number of credit-related variables. That is, the implicit rate of return on home ownership depends not only upon the yield rate on housing  $(r_h)$  and cost-of-credit  $(c_d)$ , but also on the change in the cost-of-credit  $(c_d)$ , the rate of discount (r), and the loan-to-value ratio  $(\alpha)$ . This implicit rate on housing is:

$$r_{h}^{*} = r_{h} - \alpha(r)(c_{d}) + \alpha(c_{d}) + (1 - \alpha)r,$$
 [3.15]

which is obtained from the first-order maximizing condition of the portfolio with respect to housing.

If we examine the expression [3.15] above, we see that a change in

the cost-of-credit,  $\dot{c}_d$ , will have a positive effect on the implicit rate of return on housing, but that the level of the cost-of-credit will have a negative effect.

From the Euler equations, it is evident that a change in the housing stock ( $\dot{H}$ ) will depend upon the implicit rate of return on housing ( $r_h^*$ ). Also, from the second constraint in the Lagrangian expression, the amount of debt accumulated (D) is some proportion ( $\alpha$ ) of the change in housing. Therefore, the amount of debt accumulated during period t will be a function of the change in the cost-of-credit ( $\dot{c}_d$ ).

The ratio of the implicit rates of return on the assets:

Rates of substitution (RS) =  $(\partial F/\partial H)/(\partial F/\partial j) = r_h */r_j *$ , where j = b, e, and s,

the constraints:

 $D = \alpha \dot{H}$ , and  $(1 - \alpha)\dot{H} = R + W - C - E - S - B - R_e$ 

and the conditions that:

 $r_h^* = r_h - r_\alpha c_d^+ + \alpha (c_d^-) + (1 - \alpha)r$  and  $r_j^* = r_j^+ r$ , where j = b, e, and s, provide a model of household portfolio behavior.

#### C. Demand for Mortgage Credit

By solving the first-order conditions of maximization for the amount of debt, D(t), accumulated by the household in any period, we can obtain the demand for mortgage credit as a function of the implicit rates of return on housing, bonds, savings, and equities, and the component of wages and salaries not used for consumption expenditures plus the return on the portfolio held.

$$D(t) = f_{1}(r_{h}^{*}, r_{e}^{*}, r_{s}^{*}, r_{b}^{*}, W - C + R - R_{e}^{+} + u.$$
[3.16]

If the second constraint, equation 3.9 above, is rearranged and solved for  $W - C + R - R_{c}$ , then the results are:

 $W - C + R - R_{a} = (1 - \alpha)\dot{H} + \dot{B} + \dot{E} + \dot{S}.$ 

This expression says that any change in the stock of assets less liabilities is due to the flow of funds that is available for saving purposes, i.e., the return on the portfolio plus the wages and salaries not used for consumption purposes munus the amortized payment of past debts. Therefore, a change in the value of the assets can be represented by the change in net worth. That is, the value of the assets minus the value of the liabilities is equal to net worth. Since the above expression is presented in terms of first differences with respect to time, the variables  $(1 - \alpha)\dot{H}$ ,  $\dot{E}$ ,  $\dot{S}$ , and  $\dot{B}$ , can be replaced by the change in net worth,  $\dot{NW}$ .

 $W - C + R - R_e = NW.$ 

The net receipts of the portfolio, R, include both realized and unrealized gains in the assets held by the household. Thus, net receipts will be caused not only by the acquisition of additional assets but also changes in the value of assets presently held.

The demand for mortgage credit depends heavily upon the change in the implicit rates of return that are obtained from the assets held in the portfolio. But the change in the implicit rate of return on housing depends upon the terms of mortgage credit, since housing is the only asset included in the model that may be purchased with the use of debt financing. Therefore, a household's demand for conventional mortgage credit will be:

$$D = f_2(r_h, c_d, \alpha, r_b^*, NW) + u_2.$$
 [3.17]

#### IV. SUPPLY OF MORTGAGE CREDIT

#### A. Introduction

The supply of mortgage credit from S & L's depends primarily upon their willingness to purchase conventional mortgages for the financing of single-family, owner-occupied homes. This statement does not imply that the ability of S & L's to lend is solely determined by their own objective functions and the legalities and regulations that constrain those functions. What this suggests is that the mortgage credit supplied by S & L's is also constrained by the amount of funds at its disposal to purchase mortgages, i.e., savings deposits, FHLB advances, etc. Because S & L's are primarily mortgage lending institutions, "their investments are limited by law, regulation and custom mainly to loans secured by residential real estate, and particularly by the single-family, owneroccupied home" (29, p. 76). In 1970, mortgage loans secured by singlefamily homes made up 75.8 percent of the dollar volume of loans issued by Federal Savings and Loan Insurance Corporation (FSLIC) insured S & L's. Mortgage loans held by S & L's totaled \$150.6 billion or 98.8 percent of the total loan portfolio.<sup>2</sup> As a result, it is evident that the decision by the S & L's to purchase additional mortgages is constrained by variables over which they have little or no control.

S & L's deal primarily in mortgages, which makes for a simple balance sheet (Table 1). But this does not mean that mortgages are the only asset in their portfolios. Some of the other assets that appear in S & L balance

<sup>&</sup>lt;sup>2</sup>The data presented throughout this section was obtained from the <u>Savings and Loan Fact Book</u>, 1971 (29).

Item	Amount (Millions)	Percentage of Total
ASSETS :		
Mortgage Loans Outstanding:		
Insured by the Federal Housing		
Administration	\$ 10,195	5.8%
Guaranteed by the Veterans		
Administration	8,507	4.8
Conventional Loans	131,860	74.7
Passbook, Home Improvement and Other		
Nonmortgage Loans	1,979	1.1
Federal Home Loan Bank Stock	1,578	00.9
Cash on Hand and in Banks	3,520	· 2.0
Regulatory Liquid İnvestments	12,037	6.8
Other Legal Investments	1,021	0.6
Building and Equipment	2,562	1.5
All Other Assets	3,315	1.9
Total Assets	\$176,574	100.0%
LIABILITIES AND RESERVES:		
Savings Deposits:	·	
Earning Regular Rate or Below	\$ 87,113	49.3%
Earning in Excess of Regular Rate	59,631	33.8
Federal Home Loan Bank Advances	10,490	5.9
Other Borrowed Money	452	0.3
Loans in Process	3,087	1.7
All Other Liabilities	3,789	2.1
General and Unallocated Reserves	12,012	6.8
Total Liabilities and Reserves	\$1 <b>76,57</b> 4	100.0%

# TABLE 1

CONDENSED STATEMENT OF CONDITION OF ALL SAVINGS AND LOAN ASSOCIATIONS AS OF DECEMBER 31, 1970<sup>a</sup>, <sup>b</sup>, <sup>c</sup>

<sup>a</sup>Percentages may not add to totals due to rounding.

<sup>b</sup>Preliminary.

<sup>C</sup>Sources: Federal Home Loan Bank Board; United States Savings and Loan League.

sheets are (1) government securities and investment and (2) cash. These two assets are held because of liquidity requirements that are placed on S & L's to insure against any unusual change in the flow of deposits out of S & L's.

Three assets, mortgage loans outstanding, regulatory liquid investments, and cash on hand and in banks, made up 94.1 percent of the total assets held by S & L's at year-end 1970. If only conventional mortgages are considered rather than all mortgage loans, the three assets still accounted for 83.5 percent of the assets held. It is evident that S & L's are very active in a specific component of the mortgage credit market. That component is conventional loans secured by single-family, owneroccupied homes.

The liabilities and reserves side of the balance sheet for S & L's year-end 1970 consists primarily of three items, savings deposits and Federal Home Loan Bank advances in the liability section, and general unallocated reserves in the reserves section. Similar to any firm or financial institution, the claims against S & L's is the savings deposits. At year-end 1970, these deposits accounted for 89.2 percent of the total liabilities. As a result, the savings deposits are the primary source of funds for extending mortgage credit. Any control that S & L's may have over the fluctuations in these deposits will provide additional security with respect to the liquidity obligations that must be met during the time period. The rate of interest paid on savings deposits is the most significant form of control over these deposits that the S & L's have, even though regulations have limited the rates that S & L's can pay.

The flow of funds in and out of S & L's is still susceptible to economic conditions, in spite of the FHLBB attempts to stabilize this flow, or at least to dampen the effect that a sudden change in the S & L's liabilities (decrease) will have on the asset side of the portfolio. The FHLB system performs this function by making loanable funds available to S & L's in the form of advances. When interest rates are high relative to the rate paid on deposits, savings move out of S & L's into investment alternatives. Because the S & L's portfolio consists primarily of mortgages, which are long-term contracts, whereas savings deposits at S & L's can be very short-term in nature, an unexpected flow of deposits from the S & L's could cause a liquidity problem. The dilemma of the S & L industry is the imbalance between the time elements of assets and liabilities. Since most of the funds used by S & L's to purchase mortgages come from the deposits placed in these associations, the quantity and stability of deposits are important considerations.

Advances from the Federal Home Loan Bank System is another liability listed in the balance sheet of S & L's. It accounted for 6.4 percent of the total liabilities of S & L's at year-end 1970, making it the second largest liability of S & L's. Advances are loans which are subject to a wide range of terms and conditions that are made to member associations by the FHLBB. They may be for a short or a long term, secured or unsecured, and amortized or single-payment. At year-end 1970, 71 percent of the advances to members were short-term and 98.2 percent were secured.

The primary sources of funds for S & L's to extend credit are the savings deposits and the FHLBB advances. Together, these liabilities accounted for 89 percent of the assets held at year-end 1970.

The general and unallocated reserves of the S & L's are set aside to protect depositors against any loss that may be incurred on the asset side of the balance sheet. The unallocated portion of reserves (surplus) may be used to pay interest on savings accounts in the event that current income is not sufficient to meet this expense (10).

#### B. Maximizing the S & L's Portfolio

S & L's can be federally chartered or state chartered. The federally chartered associations are mutually owned. The state chartered associations can be mutually owned or have some form of permanent stock ownership. At the end of 1970, 87.6 percent of the associations were mutual institutions and these associations held 79.3 percent of the aggregate assets of the savings and loan industry. The mutual associations outdistant the permanent stock associations in terms of numbers and assets held, 9 to 1 and 4 to 1, respectively. Therefore, in accordance with the mutual association's attempt to maintain high dividends to its owners (shareholders), who in this case are the depositors, and to provide the maximum security for the association, profits will be defined as the dividend paid on capital, general reserves, and surplus. The dividend paid to the shareholders are fixed by the advertised interest rate on savings deposits, which has an upper limit placed upon it by the Federal Home Loan Bank Board. General reserves are usually held in accordance with the regulations of the Federal Savings and Loan Insurance Corporation. Therefore, to obtain a maximization of profits by S & L's, the only variable that can be maximized is the unallocated portion of reserves or surplus ( $R_u$ ). Since this is the variable that is maximized by the

permanent stock association, we have thus an objective function that is relevant for either type of S & L organization, stock or mutual.

Since savings deposits and FHLB advances are the primary liabilities of S & L's, we will assume that these are the only sources of funds available to the S & L's when they attempt to maximize their profits. Because of the regulations that surround the S & L's portfolio composition, the only assets assumed to exist in our model will be conventional mortgages, cash and government securities. The function that will be maximized by the association will then be:

$$R_{u} = i_{m}(t)N(t) + i_{m}(s)M(t) + i_{g}(t)G(t) - i_{f}(t)A(t) - i_{s}(t)S_{L}(t) - F,^{3}$$

where  $R_u$  is the undistributed reserves;  $i_m(t)$  is the interest rate charged on mortgages in period t; N(t) is the dollar value of mortgages accumulated by the S & L during the period t;  $i_m(s)$  is the average interest rate charged on past mortgages; M(t) is the amount of past mortgages outstanding in period t; M(t) = M(s) -  $R_e$ , where  $R_e$  is outstanding mortgage credit repaid in period t;  $i_g(t)$  is the average interest rate on government securities; G(t) is the dollar value of government securities held by the S & L;  $i_f(t)$  is the average interest rate of FHLB advances; A(t) is the dollar value of advances;  $i_s(t)$  is the rate of interest paid on ordinary passbook savings deposits;  $S_L(t)$  is the dollar value of savings deposited at the S & L; and F is the fixed cost associated with S & L operations during the period.

The constraints of this function are the liquidity requirement, the funds available for mortgage purchases by the S & L, and the limit on advances from the FHLB.

<sup>3</sup>A glossary of terms is provided at the end of this study.

The first constraint, the liquidity requirement, states that the amount of cash on hand (C) and government securities (G) held by the S & L for liquidity purposes is some percentage ( $\delta$ ) of the total savings accounts held by the savings and loan. The constraint will be:

 $C + G = \delta S_{T}$ 

If the liquidity needs of the S & L increase during a period of higher interest rates, the liquidation of government securities could result in capital losses. Therefore, to insure against the possibility of such losses, a certain portion of the liquidity reserves are held in the form of cash (20). The second constraint is:

 $C/(C + G) = \beta_{0}$ , which can be reduced to,

 $C = (\beta_0 / (1 - \beta_0))G.$ 

If we substitute the above expression, the result is:

 $(1/(1 - \beta_0))G = \delta S_{T_1}$ 

Because of the legal restrictions on substitution among assets, loans secured by real estate is the S & L's primary asset; the S & L is constrained by the funds it has available for investment into that asset. The third constraint can be expressed as:

 $N = (1 - \delta)\dot{S}_{1} + \dot{A} + R_{e},$ 

where  $(1 - \delta)$  is the portion of savings available for mortgage investment after liquidity requirements are met; S is the net change in the level of savings at the S & L; A is the net change in FHLB advances; and R<sub>e</sub> is the amortized repayments from past mortgage loans.

The last constraint is the policy condition placed on the S & L by the Federal Home Loan Bank Board with respect to advances. The amount of

advances that the S & L can receive from the FHLB has an upper limit established as a percent of the claims against the association. Since the only claims considered in this model that would qualify is the savings accounts, the constraint is:

 $A \leq \gamma S_{T.}$ 

To maximize the discounted flow of net receipts from the portfolio subject to the constraints presented above, consider the Lagrangian expression:

$$J = \int_{O} \left\{ e^{-rt} [i_{m}(t)N(t) + i_{m}(s)M(t) + i_{g}G + i_{f}A - i_{s}S_{L} - F] + \lambda_{O}(t) [(1/(1 - \beta_{O})G - \delta S_{L} + \lambda_{1}(t)[N - (1 - \delta)\dot{S}_{L} - \dot{A} - R_{e}] + \lambda_{2}(t)[A - rS_{L}] \right\} dt.$$
[4.1]

Let everything within the braces be represented by f(t), thus simplifying the expression above to:

$$J = \int_{0}^{\infty} f(t) dt.$$

The first-order conditions necessary for a maximum of portfolio net return are:

$$\frac{\partial f(t)}{\partial N} = i_m e^{-rt} + \lambda_1(t) = 0.$$
[4.2]

$$\frac{\partial f(t)}{\partial G} = i_g e^{-rt} + (1/(1 - \beta_0)) \lambda_0(t) = 0.$$
 [4.3]

$$\frac{\partial f(t)}{\partial A} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial A} \right) = i_{f} e^{-rt} + \lambda_{2}(t) - \frac{d}{dt} \left[ -\lambda_{1}(t) \right] = 0.$$
[4.4]

$$\frac{\partial f(t)}{\partial S_{L}} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial S_{L}} \right) = i_{s} e^{-rt} - \delta \lambda_{o}(t) - \gamma \lambda_{2}(t)$$
$$- \frac{d}{dt} \left[ -(1 - \delta) \lambda_{1} \right] = 0. \qquad [4.5]$$

$$\frac{\partial f(t)}{\partial \lambda_{o}(t)} = [1/(1 - \beta_{o})]G - \delta S_{L} = 0.$$
[4.6]

$$\frac{\partial f(t)}{\partial \lambda_{1}(t)} = N - (1 - \delta)\dot{S}_{L} - \dot{A} - R_{e} = 0.$$

$$\frac{\partial f(t)}{\partial \lambda_{e}(t)} = rS_{L} - A \ge 0.$$
[4.7]
[4.7]

From the first-order conditions, the Lagrangian multipliers are,  $\lambda_0(t) = -(1 - \beta_0)i_g e^{-rt}$ , and  $\lambda_1(t) = -i_m e^{-rt}$ . The time derivative of the second multiplier is  $d/dt(\lambda_1) = ri_m e^{-rt} - e^{-rt}i_m$ . As a result, if the strict equality holds in the constraint on advances, then the multiplier,  $\lambda_2(t)$ , will be equal to  $ri_m e^{-rt} - e^{-rt}i_m - i_f e^{-rt}$ . On the other hand, the other economically feasible solution would be when the strict inequality holds. In that case, from the Kuhn-Tucker conditions, the multiplier is equal to zero ( $\lambda_2(t) = 0$ ). This implies that when the S & L's are borrowing from the FHLB at a level less than the maximum allowed by FHLB Board policy, then the shadow cost of these loans ( $\lambda_2$ ) is not relevant to the decision process. It is only when the constraint is satisfied that the multiplier is relevant to the first order conditions with respect to savings and advances.

After making the proper substitutions into the first-order condition with respect to savings deposits  $(S_{T})$ , the results are:

$$\frac{\partial f(t)}{\partial S_{L}} - \frac{d}{dt} \left( \frac{\partial f(t)}{\partial S_{L}} \right) = e^{-rt} \left[ -i_{s} + \delta (1-\beta_{o})i_{g} + (1-\delta)(ri_{m} - i_{m}) \right] = 0,$$

assuming that the constraint on advances is not satisfied. For the above expression to equal zero, the quantity inside the brackets must equal zero, which means that:

$$i_{s} - \delta(1 - \beta_{o})i_{g} - (1 - \delta)(ri_{m} - i_{m}) = 0,$$
 [4.9]

or the marginal cost of an additional amount of savings flow into the S & L must equal its marginal revenue to insure that profits are being maximized.

The marginal cost of an additional unit of savings deposited at the S & L is  $i_s + (1-\delta)i_m$ . This expression states that when the S & L increases, the cost of those additional units of savings will include the interest paid to the depositors and the opportunity cost of mortgage yield rate changes.

The marginal revenue is  $(1 - \beta_0)i_g + (1 - \delta)ri_m$ . If we consider the situation when savings at the S & L increase, the marginal revenue associated with that increase in savings would be the revenue received from that portion of the savings that is used to satisfy the liquidity requirement in the form of additional government securities plus the revenue from the purchase of additional mortgages at the present mortgage rate.

The analysis presented of the marginal cost equal to the marginal revenue condition of profit maximization on behalf of the S & L, suggests the importance of savings deposits to the operation of the financial intermediary. As a result, any policy decisions that are made by federal or state agencies that could have a direct or indirect impact upon the level of those deposits within the association are subject to analysis by the S & L industry. In this section, like the previous section on the demand for mortgage credit, we will consider the discretionary policy effects of the Federal Reserve System, but an additional policy element will be injected by including the ability of the S & L to manage its liabilities through the use of FHLB advances.

### C. Supply of Mortgage Credit

The first-order condition of profit maximization with respect to savings deposits (S<sub>L</sub>), that yields the marginal (factor) cost equal marginal revenue (product) condition is:

$$i_s = \delta(1 - \beta_0)i_g + (1 - \delta)(ri_m - i_m),$$

the first-order condition of profit maximization with respect to FHLB advances (A), is:

$$i_f = ri_m - i_m,$$

and the constraint:

$$N = (1 - \delta)\dot{S}_{L} + \dot{A} + R_{e},$$

represent the conditions of traditional maximizing behavior sufficient to derive a supply function for gross flows of mortgage credit.

If these conditions are solved for the gross mortgage credit supplied (N(t)), the following expression results:

$$N(t) = f_3(S_L, A, R_e, i_m, i_s, i_f, i_m) + u$$
 [4.10]

The independent variables of the supply of mortgage credit from a savings and loan association are the net flow of savings, the net flow of FHLB advances, repayments, the rate of interest on mortgages, the cost of the two sources of funds, savings deposits and FHLB advances, and the change in the rate on mortgages. Since the profit maximizing conditions of the model above require a comparison of the costs of the two sources of funds with the mortgage rate, it appears possible to use the difference between the weighted average of the costs and the mortgage rate. The weights will be determined by the portion of that particular liability used for mortgage purchases. The comparison will then be:

 $i_m - (a_0 i_f + a_1 i_s)$  where  $a_0 = A/M$  and  $a_1 = (1 - \delta)S_L/M$ . To simplify this comparison even more,  $i_c$  will be substituted for  $a_0 i_f$  + a<sub>1</sub>i<sub>s</sub>, thus, the comparison becomes:

im - ic.

The gross supply of mortgage credit, assuming that it can be expressed in stochastic form, will be expressed as:

$$N = f_4(\dot{S}_L, \dot{A}, \dot{m} - \dot{c}, R_e, \dot{m}) + u .$$
 [4.11]

## D. Aggregation

Before proceeding any further in the discussion of the mortgage credit market, it is necessary that the model of the demand by a household and the supply of S & L be transformed to a model of the entire market. The model will be expanded to include all households that demand credit from S & L's and all S & L's that supply mortgage credit. To perform such an aggregation, it is assumed that the households that comprise the demand side of the mortgage credit market and the S & L's on the supply side of the market are homogeneous and consistent with the descriptions and qualifications presented above. This assumption will not alter the variables that are included in the supply and demand functions but will facilitate the empirical analysis by allowing the use of aggregate time-series data.

### V. THE ADJUSTMENT MECHANISM

The two sides of the mortgage credit market have been presented. It is not necessary to introduce the relationships that are assumed to exist between the different elements of the mortgage market. Needless to say, not only do recursive relationships exist but also simultaneous relationships that cannot be ignored if a complete analysis of this portion of the mortgage market is to be presented.

In equilibrium the mortgage credit demand will be equal to the credit supply. This equilibrium relationship can be used in general to derive the equilibrium mortgage rate. However, it is often suggested that such an equilibrium condition does not exist in the mortgage credit market but that adjustments take place in the market such that the mortgage rate moves toward the equilibrium rate of interest. As a result, to analyze the disequilibrium situation, an adjustment mechanism must be included in our system of equations.

There are a number of alternative methods that have been presented to handle the adjustment problem (14), (15), (16), all of which have different underlying assumptions about the mechanics of the adjustment. There are three assumptions necessary to provide an interpretation of the mechanism used in this paper. First, excess demand for mortgage credit is the only situation that is assumed to exist in the market, i.e., it is impossible to have supply greater than demand. Second, an adjustment in the mortgage rate of interest is a result of current gross mortgage credit demand moving into line with predetermined gross mortgage credit supplied. Third, the mortgage credit market is cleared by the end of the quarter, even though

credit rationing by the S & L's is assumed to exist. This is due to any unexpected changes in the predetermined supply of mortgage credit that will occur as a result of unexpected and often uncontrollable changes in the net savings flow into the S & L's. The adjustment of the interest rate on mortgages will depend upon the change in the stock of mortgage credit. That is, the change in mortgage credit outstanding,  $\dot{M}(t)$ , is equal to the desired level of mortgages held by households, M\*, minus the actual level of mortgages held at the beginning of the period,  $M_{t-1}$ . The first task is to determine the level of mortgages that households desire to hold. As stated above, it is the desired stock of mortgages held that is used as the basis for deriving the demand-for-mortgage-credit expression, there fore, the desired stock of mortgage credit outstanding is equal to the actual level plus gross demand minus repayments (M\* =  $M_{t-1} + D(t) - \theta L(s)$ ),<sup>4</sup>

The change in the stock of mortgage credit outstanding is:

$$\dot{M}(t) = M^* - M_{t-1},$$
  
but  $M^* = M_{t-1} + D(t) - \Theta L(s),$   
therefore,  $\dot{M}(t) = M_{t-1} + D(t) - \Theta L(s) - M_{t-1},$   
which can be reduced to,

$$\dot{M}(t) = D(t) - \theta L(s).$$
 [5.1]

As a result, the adjustment mechanism can be expressed as:

$$\dot{c}_{d} = \dot{i}_{m} = f_{5}(\dot{M}(t)) + u_{5}.$$
 [5.2]

As noted above, this mechanism applies not only to the change in the interest rate on mortgages  $(i_m)$  in the supply function, but also to the change in the cost-of-credit  $(c_d)$  in the demand equation. This is due to

<sup>&</sup>lt;sup>4</sup>A glossary of terms is provided at the end of this study.

the fact that the income received by the S & L's from mortgage credit extended is equal to the cost of that credit to the individuals. Therefore, the change in the cost-of-credit is equal to the change in the interest rate on mortgages  $(i_m^* = \dot{c}_d)$ .

The adjustment mechanism, as outlined above, does not take into consideration the direct effects of monetary policy. This is an important point because the change in mortgage credit outstanding may have only a minor role in determining the nominal interest rate on mortgages.

## VI. MONETARY EFFECTS

### A. Introduction

To analyze the effects of monetary policy on the mortgage credit market, the cost-of-capital effect, the wealth effect, and credit rationing must be introduced into the model. But before the channeled effects can be presented, variables must be formulated that can be used to reflect changes in discretionary monetary policy.

The first of the two variables designated to reflect policy changes is the change in the commercial banking system's potential to extend credit. This particular variable is best illustrated by the change in the monetary base, MB.<sup>5</sup> This is done for two reasons: (1) the monetary base is considered an important link between Federal Reserve monetary actions and their ultimate impact on the economy's performance by a significant body of monetary theory, and (2) because Federal Reserve Credit, most of which consists of securities held by the Federal Reserve (Fed), is the principal source of the monetary base, the monetary authorities have almost complete control over the base (1). Thus, the base reflects the monetary authorities' actions better than any other measure (Table 2).

Since securities held by the Fed are so important as a source of the monetary base, it will be important in its ability to control the uses of the monetary base. It is through open market operations, the buying and selling of securities, that the Fed is able to control the source and,

<sup>5</sup>A glossary of terms is provided at the end of this study.

## TABLE 2

# CALCULATION OF THE SOURCE BASE - JUNE 1968 Monthly Averages of Daily Figures (Millions of Dollars)

Federal Reserve Credit:	
Holdings of Securities	+51,396 <sup>b</sup>
Discounts & Advances	+ 705
Float	+ 1,712
Gold Stock	+10,369
Treasury Currency Outstanding	+ 6,744
Treasury Deposits at Federal Reserve	- 960
Treasury Cash Holdings	- 973
Other Deposits and Other	•
Federal Reserve Accounts	- 177
Source Base	68,816

### Uses of Base

Sources of Base

+21,350
+ 5,566
+41,900
<b>68,8</b> 16

a Data are not adjusted for seasonal variation.

b Includes acceptances of \$90 million not shown separately.

Source: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin. The sources and uses of the base are a rearrangement of data contained in the first table appearing in the Financial and Business Statistics section of the Bulletin -- "Member Bank Reserves, Federal Reserve Bank Credit, and Related Items." therefore, has some control over the uses of the base. If the Fed's open market policy is to purchase securities, then this enables Federal Reserve Credit to increase on the sources side of the base equation, and either currency or member bank deposits to increase on the uses side (1). Even though the source base makes up the most significant portion of the monetary base, it is necessary to provide further adjustments to the source base in order to obtain the monetary base desired for analytical use.

"Because of changes in laws and regulations and in the distribution of deposits among banks subject to different regulations, adjustments must be made on the source base in order to maintain comparability over time" (1, p. 8). These adjustments in the reserve positions of the banking system will be applied to the source and expressed in dollar amounts which are positive when average reserve requirements fall and negative when they rise (1).

As a result, the monetary base not only takes into consideration open market operations through its effect on the source side of the monetary base equation but also the changes in the required reserve position of commercial banks by including the reserve adjustments. Even after taking into consideration the adjustments made in the monetary base to offset the short-run fluctuations in the total reserve position of the banking system because of changes in items that are not directly controlled such as float and currency in circulation, the monetary base still provides a good indicator of monetary policy decisions.

The second variable is the change in the discount rate,  $r_d$ , which is the third discretionary tool available to the Fed. This variable is used as a proxy for the change, or announcing the change, in the cost-of-credit.

When the Board of Governors approve a change in the discount rate, it changes the cost to member banks of borrowing from Reserve banks, thus reflecting possible changes in credit costs (2).

Most academic economists agree that if the discount rate does have any affect on the economy, it is probably the announcement effect. However, in the 1954 "Report on the Discount Mechanism" (3), it was indicated that periodic revisions in the discount rate must occur to adjust the cost of borrowing from the central bank to fall in line with market rates. The results of such "technical adjustment" would cause unclear, unnecessary and often perverse announcement effects. Because it is difficult to separate the announcement effect changes in the discount rate from the technical adjustments, it is assumed that any change in the discount rate, r, is announcing credit policy at the Fed. Also, even though discounts and advances are included under Federal Reserve Credit as a source of the monetary base, using the change in the discount rate may provide some information about the movement of the interest rate on mortgages with respect to other interest rates in the economy, i.e., the time lag needed in the mortgage credit market to allow the mortgage rate to adjust to policy changes reflected in the discount rate.

## B. Monetary Effects on Demand

When the two variables used to reflect mometary policy, MB and  $r_d$ , are included on the demand side of mortgage credit, all three discretionary tools of monetary policy are taken into consideration. It is the channeled effects of these two variables that provide the necessary conditions to measure the effect of monetary policy on the demand for mortgage credit.

Monetary policy is assumed to operate through two channels on the demand side of the market. The first effect is the change in the return, on the asset portfolio of the households without altering the composition of that portfolio. This is the wealth effect. The wealth effect will be expressed by making the change in net worth, NW, which appears in the demand function above, dependent upon wages and salaries, gross flow of mortgage credit supplied in the previous period, and the two variables of monetary policy--the change in the monetary base and the change in the discount rate, MB and  $\dot{\mathbf{r}}_{d}$  respectively.

$$\dot{Nw} = f_6(W, N_{-1}, \dot{MB}, \dot{r}_d) + u_6$$
 [6.1]

The second effect is the cost-of-capital effect. This particular effect will fall most significantly upon the cost-of-credit  $(\dot{c}_d = \dot{i}_m)$ . As a result, the cost-of-capital effect will be included in the adjustment mechanism to express the fact that a change in the interest rate on mortgages can be a result of monetary policy changes as well as a result of the demand for mortgage credit adjusting to supply, i.e., shifts in both the demand and supply functions are as possible as movements along the demand curve with respect to the interest rate on mortgages. Expressed in functional form, this channeled effect can be represented by:

$$\dot{c}_{d} = \dot{i}_{m} = f_{7}((N - \theta L(s)), \dot{r}_{d}, \dot{M}B) + u_{7}$$
 [6.2]

## C. Monetary Effects on Supply

Savings and loan associations are faced with a task familiar to savings intermediaries, the problem of managing their liabilities. The S & L's are basically restricted to the type of assets they may hold and

are therefore primarily concerned with the stability and availability of their liabilities. This problem can be illustrated by the situation where the S & L's experience a net flow of deposits out of their institutions into other savings, consumption, or investment alternatives. Because of the long-term nature of the assets they hold, it is difficult for the S & L's to adjust their liquidity position without borrowing from the FHLB. Advances from the FHLB, however, are limited by the level of deposits held by the S & L's as well as the higher cost faced by the FHLB to obtain funds to make the advances. This will make it more costly for S & L's to obtain advances in order to secure their liquidity position. It is this type of situation that causes unusual non-price rationing of the funds that S & L's may have available for extending new loans.

One of the possible causes of a change in the level of deposits at S & L's is the effect of a change in monetary policy. The first channel through which changes in monetary policy affect the supply side of the mortgage credit market is credit rationing. This effect is felt by the customers of S & L's as a result of the limited availability of funds to the S & L due to a decrease in the level of the S & L's deposit liabilities during tight monetary control periods. This can be represented by forming an expression where the net flow of deposits into S & L's is a function of the two variables of monetary policy.

$$S_{L} = f_{8}(NW, NW_{-1}, MB, r_{d}) + u_{g}$$
 [6.3]

Also included in the above expression is the change in net worth (NW) and the change in net worth of the previous period. By including the change in net worth in the net savings flow expression, the wealth effect that is felt by those households holding asset portfolios, part or

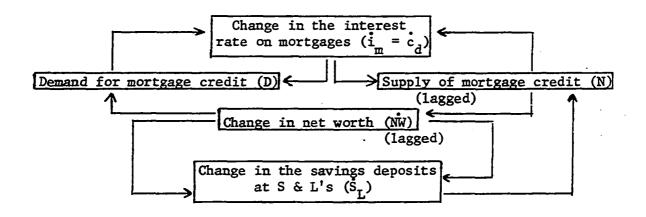
all of which consists of savings deposits at S & L's, is accounted for on the supply side of the mortgage credit market by including the effect that these changes have on net savings flow.

Since the change in the cost-of-credit is equal to the change in the interest rate on mortgages, the cost-of-capital effect can be included on the supply side because  $i_m$  is one of the variables that appears on the right hand side. This would imply that the cost-of-capital effect would also operate on the supply side.

D. The Complete Model

All the different parts of the model have been presented above. This section of the paper is designed to incorporate the parts into a complete model for empirical testing.

The model will consist of equations 3.17, 4.11, 6.1, 6.2, and 6.3, which are a demand equation, a supply equation, an equation for the change in net worth, an adjustment mechanism for interest rates, and an equation for the net flow of savings deposits with S & L's, respectively. As a result, the following flow chart diagram may facilitate the understanding of the assumed interdependencies that exist in the model.



### VII. EMPIRICAL RESULTS

### A. Introduction

The statistical techniques that are used to obtain the "best" estimates of the coefficients of the variables that appear in the model are (1) ordinary least-squares (OLS) to derive first estimates before other techniques are used, (2) ordinary least-squares on the transformed data to circumvent the problem of autocorrelation (generalized leastsquares, or GLS), and (3) two-stage least-squares (TSLS) in an attempt to account for the simultaneity bias that may be present within the model. The primary reason for the use of more than one statistical method in obtaining coefficient estimates is to provide a basis for comparison since it is very difficult to compare these results with the statistical results obtained in other studies of the mortgage credit market. This is due primarily to the fact that past mortgage credit models have been designed as stock adjustment models. As a result, most of the models have included the lagged endogenous variable as one of the predetermined variables (16), (23), (25).

The structural equations presented in the previous section are most informative about the restrictive hypotheses that the model is designed to consider, i.e., the channeled effects of mometary policy. Therefore, since the evaluation of the hypotheses depends upon the quality of the estimates obtained, a statistical method that will provide efficient, consistent, and unbiased estimates is the ideal. Unfortunately, this is not always possible to do. As a result, certain tradeoffs have to be

made, e.g., the estimates may be consistent and efficient but biased. It is the objective of this section to obtain the best estimates of the mortgage credit market.

B. Ordinary Least-Squares (OLS)

The first technique, ordinary least-squares, is used to derive first estimates of the coefficients that appear in the structural equations of the model. With the aid of this particular operation, it is possible to obtain information about the need for subsequent statistical methods to be applied to the model. This technique provides estimates for the parameters, "t" statistics to aid in the decision of whether the particular independent variables are significant, the corrected squared coefficient of multiple correlation ( $\overline{\mathbb{R}}$ ) which aids the researcher in determining the proportion of explained variance, and the Durbin-Watson statistic which is a test of autocorrelation of the residuals.

OLS is applied to the demand for mortgage credit. The results are:  $D = -55.18 + .78 \alpha + .216 r_{h} + 3.87 r_{b}^{*} + 12.94 \Delta c_{d} + .060 \Delta NW,^{6} [7.1]$   $(46.83) (.63) (.106)^{h} (1.49)^{b} (3.61) (.042)$   $R^{2} = .65,$  D-W = 1.23,

where the numbers in parentheses below the coefficients are their standard errors.

Although the "t statistics" for all coefficients except that of the loan-to-value ratio are significant,<sup>7</sup> concern is caused by the low  $\overline{R}^2$  and the low Durbin-Watson statistic that is obtained by the

<sup>6</sup>A glossary of terms is provided at the end of this study. <sup>7</sup>The significance level throughout this study is 10 percent.

use of ordinary least-squares. The  $\overline{R}^2$  suggests that only 65 percent of the variance can be explained by the variables that appear on the right side of the equation, therefore, a higher  $\overline{R}^2$  would be desirable. The Durbin-Watson (D-W) is 1.23. This value lies in the inconclusive range of positive correlation, i.e., the D-W is not low enough to guarantee the presence of positive correlation but it is not high enough to say conclusively that it does not exist. Since ordinary least-squares was used to derive these results, the D-W statistic is an appropriate test for autocorrelation if all the variables on the right side of the equation are strictly exogenous (temporarily ignoring the simultaneity of the model).

To correct for possible autocorrelation of the residuals, the data is transformed:

$$X_{it}^{*} = X_{it} - \rho X_{it-1}, \quad Y_{t}^{*} = Y_{t} - \rho Y_{t-1}, \quad X_{i1}^{*} = \sqrt{(1 - \rho^{2})} X_{i1},$$
  
and  $Y_{1}^{*} = \sqrt{(1 - \rho^{2})} Y_{1},$ 

where  $X_{it}$  are the independent variables at time t,  $Y_t$  is the dependent variable at time t,  $X_{i1}$  are the independent variables at time period 0,  $Y_1$  is the dependent variable at time period 0, and  $\rho$  is the autocorrelation coefficient of the residuals. This transformed data is used in place of the raw data in the ordinary least-squares operation. The results are:

 $D = -46.64 + .696 \alpha + .0995 r_{h} + 3.59 r_{b}^{*} + 11.22 \Delta c_{d} + .061 \Delta NW, [7.2]$  (41.04) (.540) (0.08) h (1.27) h (2.98) d (.040)  $\overline{R}^{2} = .71$   $D-W = 1.78, \rho = .5$ 

After the transformation of the data in an attempt to adjust for

possible correlation of residuals, the results suggest a better statistical fit, but not considerably better than OLS. Now 71 percent of the variance is explained and the D-W statistic exceeds the upper limit ( $d_u = 1.74$ ) which is required for no positive autocorrelation to be present, but the coefficient for the yield rate on housing, as well as the coefficient for the loan-to-value ratio, is insignificant.

The theory of demand states that as price decreases, the quantity demanded will increase per unit of time. This suggests, at some fixed period in time, a negative relationship between price and quantity.

Since the coefficient for a change in the interest rate on mortgages ( $\Delta c_d$ ) is positive, it may at first glance suggest the wrong sign. A portion of the change in the housing stock is financed by the use of credit. For this reason, the demand for mortgage credit has included as one of its variables the change in the cost-of-credit,  $\dot{c_d}$ . The firstorder conditions of the Lagrangian expression provide the solution for the implicit rates of return on the assets held in the portfolio. Therefore, if the implicit rate of return on housing is recalled,  $r_h - \alpha rc_d$  $+ \alpha \dot{c_d} - (1 - \alpha)r$ , the relationship between the implicit return and a change in the interest rate is positive. The coefficient on  $\dot{c_d}$  was expected to be positive. This is not the relationship between the stock of mortgage credit outstanding and the interest rate on mortgages, but the relationship between the gross flow of mortgages demanded and the change in the interest rate. The signs of all the coefficients are the appropriate ones to support the hypotheses of implicit rates and portfolio analyses.

The supply of mortgage credit was also subjected to the ordinary leastsquares method, the results of which are:

$$N = -8.39 + .00032 \Delta S_{L} + .00047 \Delta A + 4.089 R_{e} + .927 (1_{m} - i_{c})$$
(1.71) (.00022) (.00034) (0.527) (.366) (.366) (.366) (.366) (.367) (.3927) [7.3]
$$\frac{2}{R} = .95$$

$$D-W = 1.053.$$

In this particular equation, all the coefficients are significant. Also,  $\frac{2}{100}$  the  $\overline{R}$  is high but the Durbin-Watson is low. As a result, the same transformation that was used on the demand equation is applied to the variables that appear in the supply function. The results are somewhat different.

$$N = -9.70 + .00024 \Delta S + .00064 \Delta A + 4.57 R + .793 \Delta i_{m}$$
(1.86) (.00016) (.00026 (0.41) e (.748) m  
+ .810 (i\_m - i\_c)  
(.520) [7.4]  

$$\frac{2}{R} = .97$$
D-W = 1.62,  $\rho = .6$ 

Not only are the "t statistics" different, making the estimated coefficient of  $\Delta i_m$  insignificant, but the D-W statistic is still within the inconclusive range. These results imply that, first, the  $\Delta i_m$  is not one of the variables considered in the decision of the S & L to obtain more mortgages. This may be due to the fact that the primary income-earning asset that S & L's can legally hold is mortgages. Therefore, regardless of what the change in the nominal mortgage rate, the S & L's hold mortgages. It is possible, however, that when changes occur in the nominal rate, the S & L's are capable of managing their liabilities to some degree in order to maximize their objective function. Under liability management, the insignificance of  $\Delta i_m$  is a contradiction. Second, the D-W statistic within the inconclusive range implies that the autocorrelation of the residuals, if it does exist, may not necessarily be of the first-order type suggested by the transformation that is used on the data.

Ordinary least-squares was also applied to the other equations in the model. The first is that of the change in the net worth of the households. The results are:

$$\Delta NW = -27.49 - 9.243 \Delta r_{d} + 10.78 \Delta MB + .0966 W - 1.0288 N_{(10.07)} (5.318) (4.33) (.0217) (.5290)^{-1} [7.5]$$

$$\frac{2}{R} = .71$$

$$D-W = 1.68.$$

In this particular equation, all the estimated coefficients are significant. The  $\overline{\mathbb{R}}^2$  is acceptable, but the D-W statistic is within the inconclusive range. This latter result implies that the null hypothesis of no positive autocorrelation of the residuals can not be accepted, but the alternative cannot be accepted either. "The test is inconclusive and further observations would ideally be required" (17, p. 192). From the empirical results on the supply and demand equations, when the D-W was in the inconclusive range, the transformed data did not significantly improve the statistical results. Therefore, the generalized least-squares procedure will not be used on the change in net worth equation, the net savings inflow equation, and the adjustment mechanism.

The change in the stock of savings at S & L's (the net flow of savings deposits) is estimated as follows:

$$\Delta S_{L} = -951.603 - 43.458 \Delta NW + 103.26 \Delta NW - 1 + 1780.75 \Delta MB (664.205) (28.541) (28.11) - 1 (792.32) - 2259.73 \Delta r_{d}, (843.12) [7.6]$$

$$\frac{2}{R} = .73$$
  
D-W = 2.18

Again, the estimated equation is acceptable with the appropriate signs on the variables. However, in this particular equation we have included not only the change in net worth but also the lagged change in net worth. This implies that the initial effect of a change in net worth on net savings at S & L's is negative but after a lag of one period, the effect is positive.

OLS is also applied to the adjustment mechanism of the mortgage credit market, the change in the interest rate on mortgages.

$$\Delta c_{d} = \Delta i_{m} = .230 + .0398 \Delta M_{t} + .426 \Delta r_{d} - .2092 \Delta MB,$$

$$(.0759) (.0351) + .0847 + .0000 (.0711)$$

$$\begin{bmatrix} 7.7 \end{bmatrix}$$

$$R^{2} = .65$$

$$D-W = 1.42.$$

In this particular equation, all the monetary variables are significant, whereas, the change in mortgage credit outstanding is insignificant.

C. Estimation of the Simultaneous Model

The statistical methods used above were ordinary least-squares and generalized least-squares, but neither of these techniques can take into consideration the simultaneous relationships that exist in this market. As a result, a third statistical <u>modus operandi</u>, two-stage least-squares (TSLS) is used. There are alternative methods available to handle the simultaneous problem, such as indirect least-squares, but since the system of equations is overidentified, an appropriate method to use is two-stage least-squares. The structural equations are the most informative about the hypotheses for which the model is developed, but economists sometimes ignore the fact that the conditional equations of the reduced form provide the stepping stone of estimating the parameters of the structural equations. Therefore, because of the important role played in the simultaneous equation process by the coefficients of the reduced equations, I thought it worth the effort to include the reduced form equations in this section of the paper.

TSLS is a single-equation method. The reduced equations for the endogenous variables are:

$$N = \pi_{10} + \pi_{11} \Delta A + \pi_{12} R_{e} + \pi_{13} \Delta r_{d} + \pi_{14} \Delta MB + \pi_{15} W + \pi_{16} N_{-1} + \pi_{17} \alpha + \pi_{18} r_{h} + \pi_{19} r_{b}^{*} + \pi_{110}^{*} \Delta NW_{-1} + u_{1},$$

$$P = \pi_{10} + \pi_{10} \alpha + \pi_{10} r_{b} + \pi_{10} r_{b}^{*} + \pi_{10} \Delta RW_{-1} + u_{1},$$

$$P = \pi_{10} + \pi_{10} \alpha + \pi_{10} r_{b} + \pi_{10} r_{b}^{*} + \pi_{10} r_{b} + \pi_{10} r_{b$$

$$+ \pi_{27} W + \pi_{28} + R_{e} + \pi_{29} \Delta A + \Delta N W_{1} + u_{2}, \qquad [7.9]$$

$$\Delta c_{d} = \pi_{30} + \pi_{31} \Delta r_{d} + \pi_{32} \Delta MB + \pi_{33} r_{h} + \pi_{34} r_{b}^{*} + \pi_{35} r_{e}^{*} + \pi_{36} \Delta A + \pi_{37} \Delta NW_{-1} + \pi_{38} N_{-1} + u_{3}^{*}, \qquad [7.10]$$

$$\Delta NW = \pi_{40} + \pi_{41} \Delta r_{d} + \pi_{42} \Delta MB + \pi_{43} W + \pi_{44} N_{-1} + u_{4}, \qquad [7.11]$$
  
$$\Delta S_{T} = \pi_{50} + \pi_{51} \Delta r_{d} + \pi_{50} \Delta MB + \pi_{52} \Delta NW + \pi_{54} W + \pi_{55} N_{d}$$

$$+ u_5$$
. [7.12]

The reduced equation for the change in net worth, equation [7.11], is the same as the ordinary least-squares equation because there exist no endogenous variables on the right hand side of the equation. This equation is therefore a recursive equation and does not require the more sophisticated estimation technique of TSLS. The advantage of including a recursive equation within the system is that it provides more exogenous variables for identification purposes.

In the remainder of the equations, [7.8], [7.9], [7.10], and [7.12], the simultaneous equation technique of TSLS is required. When the ordinary least-squares method is applied to a simultaneous-equation system, the estimators are biased and inconsistent. The TSLS method does not necessarily eliminate the bias but the estimators obtained with this method "have the desirable large-sample property of consistency" (17, p. 275).

The two-stage least-squares technique when applied to the model provides the following estimates of the structural equation:

 $N = -07.98 + 4.328 \Delta i_{m} + 0.000546 \Delta s_{L} + 0.563 (i_{m} - i_{c}) \\ (01.84) (2.278) (0.000318) (0.454)$ + 0.000497  $\Delta A$  + 03.97 R (0.000355) (00.538)[7.13] 2  $\overline{R} = .93$ D-W = 1.85,  $D = -64.6 + 0.089 \text{ ANW} + 14.45 \text{ Arg} + 3.84 \text{ r}_{b}^{*} + 0.192 \text{ r}_{b}$ (55.5) (0.056) (5.29) (1.98) (0.111) [7.14]  $+ 9.092 \alpha$ (0.749)2  $\overline{R} = .63$ D-W = 1.28.  $\Delta t_{m} = \Delta c_{d} = 0.159 + 0.1006 \Delta M_{t} + .4203 \Delta r_{d} - .209 \Delta MB (.0735) (.0720) t (.0815) (.065)$ [7.15] $\overline{R}^2 = .68$ D-W = 1.43,  $\Delta NW = -27.49 - 9.24 \Delta r_{d} + 10.78 \Delta MB + .0967 W - 1.0289 N_{(10.07)} (5.32) (4.33) (.0217) (.5290)^{-1}$ [7.16]

$$\frac{2}{R} = .71 
D-W = 1.68 
\Delta S_{L} = -717.2 - 2607.11 \Delta r_{d} + 1985.45 \Delta MB - 76.67 \Delta NW 
(750.9) (982.5) (838.56) (52.98) 
+ 123.05 \Delta NW_{-1} 
(39.08) [7.17] 
 $\frac{2}{R} = .71 
D-W = 2.55.$$$

If each equation is considered separately, it is evident from the above that in most cases the simultaneous technique provides acceptable results. However, a caution to accepting the results is necessary. There are problems that can be overcome by the use of the TSLS technique but other problems still remain. In particular, three statistical problems remain, (1) autocorrelation of the residuals, (2) inadequate data, and (3) misspecification of the model.

The problem of autocorrelation of the residuals was considered above by the use of OLS on the transformed data. In this particular case, to consider the correlation problem, a trade-off must be made with simultaneity bias. A choice has to be made as to which one provides the "best" estimates for this model. This choice will depend upon the weight given to the respective statistical problems, i.e., autocorrelation of the residuals on one hand and simultaneity bias on the other.

The second problem, that of inadequate data, is most certainly present. However, this is not unfamiliar to studies of the mortgage and/or housing markets. As a result, errors in economic variables can also be a problem. "Such errors may arise because of errors of measurement, or because the measurable quantities are not quite the same conceptually as

the relevant theoretical quantities" (5, p. 251). The demand function includes three data series that are of questionable quality. They are the rate of return on housing  $(r_h)$ , the change in net worth ( $\Delta NW$ ), and the gross flow of conventional mortgage credit from S & L's (D=N).<sup>8</sup>

 $<sup>^{8}</sup>$  The glossary of terms includes a short explanation of what is used as a proxy for these variables.

### VIII. CONCLUSIONS

Credit rationing is assumed to operate primarily through the change in net savings inflow of the savings and loan industry. Because of the difficulty of measuring this particular effect, the change in net savings inflow is used as a proxy. If rationing does occur, it will be the result of a change in the liabilities of S & L's, specifically a decrease in savings flowing into intermediaries, along with the inability of the rate of interest on mortgage credit to be adjusted immediately to eliminate the credit market's excess demand. The policy variables, monetary base and discount rate, are found to be significant in the S & L's net savings flow is significant to the gross flow of mortgage credit from S & L's.

The wealth effect is assumed to operate on the household sector. This effect is the link between monetary policy on the one hand, and the level of consumption and savings, on the other. This channel of mometary policy is assumed to operate through the changes in the net worth of households. The monetary variables,  $\Delta MB$  and  $\Delta r_d$ , are significant variables in the net worth function. Likewise, the change in net worth is significant to the gross flow of mortgage credit demanded.

Monetary forces have not always been found to have such an important impact on the economy. In fact, several of the large econometric models suggest that monetary forces are unimportant, e.g., Wharton School model, the Commerce Department model, the Michigan model, and to a lesser extent the Brookings model (8). The Federal Reserve-MIT econometric model provided by the other models listed but found the monetary effects to be slow.

Differences in the importance of mometary forces can be attributed to the additional channels of policy that are included in the Federal Reserve-MIT model. The conclusion of the Fed model is that mometary policy works more slowly than fiscal policy. If the "quick-acting credit rationing and wealth effects of mometary policy" were found to be important during the first year, a more rapid operation of mometary policy would have been implied (8).

The econometric model of the mortgage credit market presented here implies that monetary policy operates in this particular market much more quickly than is implied by several other models because the credit rationing and wealth effects of monetary policy are implied to be important. From the empirical results, it is evident that the change in the monetary base and the change in the discount rate, in all equations where they appear, are not lagged but are changes in the current period.

The third channel of monetary policy is the cost of capital effect. Included in the adjustment mechanism are the two monetary variables,  $\Delta MB$ and  $\Delta r_d$ . As shown above, both of these variables are significant and have the appropriate signs. However, the change in the cost-of-credit variable that is included in the demand function has a positive sign and is highly significant. This implies that when the mortgage rate increases the demand for mortgage credit increases and when the rate declines the demand for mortgage credit falls. One explanation of the positive sign on this coefficient is that a change in the conventional mortgage rate has a positive effect on the implicit rate of return on housing. An alternative explanation is to consider the effect of inflation on this sector. Prices have not been explicitly considered in the model, but can be implied to

exist as part of the interest rate adjustment (21). The data used in the model is from the first quarter of 1965 to fourth quarter 1971. During this period, the general price level persistently increased, such that the nominal rate of interest reflected the expectations of future inflation (21). As a result, even though the nominal interest rate on mort-gages increased, the real rate may have decreased or at least increased by less than the nominal rate.

Consequently, during periods of inflation, the nominal rate of interest may include expected changes in the price level and that it is the real rate that is relevant to the demand of mortgage credit, i.e., assuming a high elasticity of housing demand with respect to the real interest rate. If inflation is expected to continue and the change in the nominal rate of interest reflects those expectations, then the prices of real assets will also be expected to increase, thus making the fixed dollar assets less attractive and the marketable real assets (e.g., housing) more attractive. As a result, the cost-of-capital effect may be slow in its effect and misleading as an indicator of monetary policy's impact on the conventional mortgage credit market during periods of inflation.

The empirical results support the hypothesis that monetary policy has a multi-channeled effect on the mortgage-credit market. It also supports the hypothesis that monetary policy has an impact on both the supply of and demand for mortgage credit. However, for short-run fluctuations in mortgage credit flows, it is concluded that only two of the three channels of monetary policy are relevant. These two channels are the credit rationing effect on the supply side and the wealth effect on the demand side.

## IX. GLOSSARY OF TERMS

Because of the number of terms and my reluctance at times to	
immediately define a term when it is presented, I include a list of these	
terms along with the appropriate definition of each.	
A - dollar value of advances held by the S & L.	
B - dollar value of bonds held in the household's portfolio.	
$D^9$ - gross debt accumulated during the period, the gross flow of	
mortgage credit demanded by the household sector.	
E - dollar value of equities held in the household's portfolio.	
F - fixed cost of operating the S & L.	
G - dollar value of government securities held by the S & L.	
H - dollar value of housing held in the household's portfolio.	
K - dollar value of the asset portfolio held by the household.	
L(s) - stock of debt held by the household.	
M - stock of mortgages outstanding to the S & L.	
MB - monetary base.	
N - gross flow of mortgage credit supplied by the S & L.	
NW <sup>10</sup> - net worth of the household.	
R - net receipts on the individual's portfolio.	
R - dollar value of amortized payments from debt accumulated in past	
periods.	
9 This is the conventional mortgage credit for single-family owner-	

<sup>9</sup>This is the conventional mortgage credit for single-family owneroccupied homes. Such a data series is not available, therefore, the total flow of mortgage credit from S&L's for home purchases was used as a proxy.

<sup>10</sup>The net worth variable includes mortgages but does not include the real asset of housing. Only financial assets are considered.

- undistributed profits of the S & L. R\_\_\_ S - dollar value of savings held in the household's portfolio. S<sub>T</sub> - dollar value of savings deposited at the S & L. W - wages and salaries of the household. - cost-of-credit. с<sub>л</sub> - average cost of funds available to the S & L. i - interest rate on Federal Home Loan Bank advances. i<sub>f</sub> - interest rate on government securities. i, - interest rate on conventional mortgage credit extended by S & L's. i\_ - interest rate on savings deposits at S & L's. i, - rate of discount (the yield rate on government 3-month bills). r - yield rate on bonds. rh  $r_{b}^{*}$  - implicit rate of return on bonds  $(r_{b}^{+} + r)$ . - discount rate at the Eed. rd - yield rate on equities. r  $r_{p}^{*}$  - implicit rate of return on equities  $(r_{p}^{*} + r)$ . r<sub>b</sub><sup>11</sup> - yield rate on housing.  $r_h^*$  - implicit rate of return on housing  $r_h - \alpha r C_d + \alpha \dot{C}_d + (1-\alpha)r$ . - yield rate on savings. r  $r_{r}$  - implicit rate of return on savings ( $r_{r}$  + r).  $X=\frac{dx}{dt}$  - continuous change in the variable over time. - loan-to-value ratio on conventional mortgages.

<sup>11</sup>The yield rate on housing was computed as the percentage distribution of personal consumption expenditures to housing services plus the rate of change in prices of houses.

- $\beta_0$  portion of liquidity reserves held in the form of cash.
- $\gamma_{\rm A}$  legal constraint on the amount of advances a S & L may obtain from the FHLB system.
- $\delta$  portion of savings held in the form of liquidity reserves.
- $\theta$  portion of past debt repaid during the period.
- △ used to illustrate a discrete change in the variable.

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### XII. APPENDIX: CALCULUS OF VARIATIONS

The calculus of variations is used to a large extent in developing the model of the mortgage credit market. To clarify the procedure used, elements of the calculus of variations are presented below.

Consider the following problem. Maximize (or minimize) the integral:  $u = \int_{t_2}^{t_1} f(t, x, \dot{x}) dt,$ 

where we denote  $\dot{x} = dx/dt$  and require that  $x(t_1) = a_1 x(t_2) = b$ , a and b given constants.

We consider now a variation in the function "f", denoted by " $\delta f$ ". We have evidently

$$\delta \mathbf{f} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \delta \mathbf{x} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \delta \mathbf{x} + \mathbf{x} \delta \mathbf{x}$$

where  $\delta x$  and  $\delta \dot{x}$  are variations of x and  $\dot{x}.$ 

Then the variation of the integral is

$$\delta u = \int_{t_1}^{t_2} (\delta f) dt = \int_{t_1}^{t_2} \left( \frac{\partial f}{\partial x} \delta x \right) dt + \int_{t_1}^{t_2} \left\{ \frac{\partial f}{\partial x} \frac{d}{dt} (\delta x) \right\} dt.$$

Now consider

$$\frac{d}{dt} \begin{bmatrix} \frac{\partial f}{\partial \dot{x}} \delta x - \int \left\{ \frac{d}{dt} \begin{pmatrix} \frac{\partial f}{\partial \dot{x}} \end{pmatrix} \delta x \right\} dt \end{bmatrix} = \frac{d}{dt} \begin{pmatrix} \frac{\partial f}{\partial \dot{x}} \delta x \end{pmatrix} - \frac{d}{dt} \begin{pmatrix} \frac{\partial f}{\partial \dot{x}} \end{pmatrix} \delta x$$
$$= \frac{d}{dt} \begin{pmatrix} \frac{\partial f}{\partial \dot{x}} \end{pmatrix} \delta x + \frac{\partial f}{\partial \dot{x}} \frac{d}{dt} (\delta x) - \frac{d}{dt} \begin{pmatrix} \frac{\partial f}{\partial \dot{x}} \end{pmatrix} \delta x = \frac{\partial f}{\partial \dot{x}} \frac{d}{dt} (\delta x).$$

where we have used the fact that the derivative of the indefinite integral is the expression under the integral sign as well as the rule of finding the derivative of a product. Hence we have, apart from a constant of integration,

$$\int \left\{ \frac{\partial f}{\partial \dot{x}} \frac{d}{dt} (\delta x) \right\} dt = \frac{\partial f}{\partial \dot{x}} \delta x - \int \left\{ \frac{d}{dt} \left| \frac{\partial f}{\partial \dot{x}} \right| \delta x \right\} dt.$$
So
$$\int_{t_{1}}^{t_{2}} \left\{ \frac{\partial f}{\partial \dot{x}} \frac{d}{dt} (\delta x) \right\} dt = \left[ \frac{\partial f}{\partial \dot{x}} \delta x \right]_{t_{1}}^{t_{2}} - \int_{t_{0}}^{t_{1}} \left\{ \frac{d}{dt} \left| \frac{\partial f}{\partial \dot{x}} \right| \delta x \right\} dt.$$

Hence the variation of the integral can be written

$$\delta \mathbf{u} = \begin{bmatrix} \frac{\partial f}{\partial \dot{\mathbf{x}}} & \delta \mathbf{x} \end{bmatrix}_{t_1}^{t_2} + \int_{t_1}^{t_2} \left\{ \frac{\partial f}{\partial \mathbf{x}} - \frac{d}{dt} \left( \frac{\partial f}{\partial \dot{\mathbf{x}}} \right) \right\} \delta \mathbf{x} dt.$$

We note that since x  $(t_1) = a$  and x  $(t_2) = b$  are given the first term on the right is zero. Apart from this condition  $\delta x$  is arbitrary. Hence for the expression under the integral to be zero, we derive the Euler equation:

$$\frac{\partial f}{\partial x} - \frac{d}{dt} \frac{\partial f}{\partial \dot{x}} = 0,$$

which is a necessary condition for a minimum or maximum of our integral.

.